

# History of Advancement of SDM-related Simulation: MCF Analysis and Design Technologies

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## 1. What is MCF?

Research on multi-core fiber (MCF) for space-division multiplexing (SDM) transmission came into full swing when the EXAT Study Group was established in 2008. MCFs are broadly divided into uncoupled-core MCF (UC-MCF) and coupled-core MCF (CC-MCF) in accordance with the distance between core centers (hereinafter referred to as core pitch). In this article, we will refer to UC-MCF as MCF.

MCF is an SDM technology that sufficiently reduces inter-core crosstalk (XT) and uses each core as an independent transmission line. It is classified into homogeneous MCF and heterogeneous MCF. The English names for these MCFs have been decided upon consultation with Professor Masataka Nakazawa and Professor Richard De La Rue<sup>[1]</sup>.

Although MCF essentially has a single mode in each core, it can accommodate multiple modes to expand the spatial multiplexing. These MCFs are either called multi-mode MCF (MM-MCF) or few-mode MCF (FM-MCF).

CC-MCF, on the other hand, utilizes each of the super-modes generated by the arrangement of multiple cores in close proximity as an independent transmission line. When the core pitch is set appropriately, the super-modes are randomly coupled to each other, and the group velocity is averaged to realize an MCF where the group delay spread (GDS) is proportional to the square root of the transmission distance. This type of MCF is called randomly coupled MCF (RC-MCF) and is suitable for long-haul transmission.

Professor Yasuo Kokubun pointed out at the EXAT Symposium held in 2008 that if the core pitch of a two-core fiber is set to about four times the core radius, the propagation constants of the even and odd modes become almost equal, making it easier to couple them<sup>[2]</sup>. Interestingly, this core pitch is almost the same as the core pitch of RC-MCF, which has been studied extensively since the mid-2010s.

## 2. MCF analysis and design technologies

Evaluating the properties of fibers initially requires performing an eigenmode analysis to know the propagation constant (eigen value) and the electromagnetic field distribution (eigen function) of each mode. In some cases, the exact solution is known, such as in round core fibers; but when the fiber structure becomes more complex, it becomes difficult to derive the exact solution, requiring numerical analysis. Although there are

various numerical solutions, the finite element method (FEM) and beam propagation method (BPM) are often used in the optical frequency range and the finite-difference time-domain method (FDTD) is often used in the radio frequency range. Currently, FEM and BPM are also essential for MCF design, but it is difficult to incorporate bending, twisting, and even random structural fluctuations. For this reason, the coupled-mode theory (CMT) and the coupled-power theory (CPT) have been developed specifically for MCF analysis.

This paper discusses CMT and CPT as MCF analysis and design technologies and outlines the progress of the development of these theories.

## 3. XT analysis of MCF

CMT has traditionally been used to evaluate coupling between optical waveguides and modes. According to this CMT, XT in MCF is expected to vary periodically in the propagation direction. In 2010, however, it was experimentally shown that XT increases in proportion to the transmission distance<sup>[3]</sup>. This suggests that the structure is randomly fluctuating in the propagation direction. The introduction of CPT then made it possible to explain this experimental result in 2010<sup>[3]</sup>.

Also, in the same year, measurement of the bending radius dependence of XT in heterogeneous MCF experimentally showed that there is a phase-matching region (PMR) and a non-PMR region (non-PMR) around the bending radius called the critical bending radius<sup>[4]</sup>. In 2011, a discrete changes model (DCM) was developed to account for random discrete changes in coupling power at phase-matching points (PMPs)<sup>[5, 6]</sup>, and it was found that XT increases with the bending radius in the PMR<sup>[5, 6]</sup>.

Despite its simplicity, DCM cannot be applied to non-PMR without PMP; thus, in 2011, a CMT was developed that takes into account bending, twisting, and random structural fluctuations<sup>[7]</sup>. As a result, it was found that XT is governed by bending in PMR and by the statistical property of the structural fluctuations (quantified with correlation length) in non-PMR<sup>[7]</sup>.

Since calculation takes a long time in CMT (requires stochastic treatment), a CPT that takes into account not only structural fluctuations but also bending and twisting was developed in 2011<sup>[7]</sup>. In this case, since the power coupling coefficient (PCC) varies in the propagation direction, the coupled-power equation is solved sequentially.

In 2012, an exponential autocorrelation function (ACF) was

introduced to derive a closed-form expression of PCC averaged over the twisting period<sup>[8]</sup> (with the correlation length as the unknown variable). This analytical model can be applied to both PMR and non-PMR and is widely used for XT analysis of MCF. In the case of MM-MCF and FM-MCF, there is a need to analyze the coupling between modes in the core; thus, a CMT capable of such analysis was developed in 2015<sup>[9]</sup>.

#### 4. GDS analysis of RC-MCF

From the mid-2010s, GDS analysis of RC-MCF came into full swing using CMT, and it was found that GDS is proportional to the square root of the transmission distance (which has been confirmed experimentally).

According to the CMT analysis, GDS is strongly dependent on the core pitch, and the core pitch that results in the minimum GDS is around 20  $\mu\text{m}$ , depending on the degree of twisting and manufacturing variability<sup>[10]</sup>. This core pitch is almost four times the core radius (core diameter of about 9  $\mu\text{m}$ ) of a silica-based single-mode fiber.

#### 5. XT analysis with PMC consideration

In 2011, it was theoretically shown that the XT distribution in MCF without polarization mode coupling (PMC) follows a chi-square distribution with two degrees of freedom, and with PMC, follows a chi-square distribution with four degrees of freedom<sup>[5, 6]</sup>. Although this XT distribution in MCF has been experimentally confirmed<sup>[6]</sup>, the dependence of XT on the birefringence correlation length and the birefringence beat length has remained undiscussed for some time.

In 2020, XT analysis with CMT was performed with no bending<sup>[11]</sup> and in 2022 with bending, including heterogeneous MCF<sup>[12]</sup> (assuming the same correlation length and the same beat length for all cores). Also in 2020, a simple analysis method for XT using polarization mode dispersion coefficient as the unknown variable was developed<sup>[13]</sup> (under the condition that the correlation length is sufficiently small compared to the beat length).

#### 6. XT analysis of MCF with BSA transmission

There has been a growing interest in bidirectional signal assignment (BSA) transmission, which can significantly reduce XT by placing signal lights with opposite propagation

directions in neighboring cores. For long-haul BSA transmission, backscattered XT and indirect XT must be evaluated, and for short-reach BSA transmission, back-reflected XT and indirect XT must be evaluated, and CPT is used for both evaluations<sup>[14]</sup>.

PCC has been implicitly considered to be symmetric, but with core-dependent loss (CDL), PCC becomes asymmetric. In 2023, it was reported that when the ACF is given by an exponential function, treating the PCC as a symmetrical quantity poses no issues within the practical ranges of CDL and correlation length<sup>[15]</sup>.

#### 7. Unresolved issues (Conclusion)

We discussed CMT and CPT as analysis and design technologies for MCF and outlined the development progress of these theories. In conclusion, below are some of the issues that need to be resolved going forward.

In particular, the anomalously large correlation lengths, the cladding-diameter dependence of XT, and the bending-radius dependence of the correlation length cannot be reasonably explained in the framework of CMT or CPT<sup>[16]</sup> (that could be resolved with a newly derived PCC<sup>[17]</sup>). Furthermore, the phenomenon where the LP mode, which is theoretically not an eigenmode, propagates as an eigenmode in FM-MCF has not yet been elucidated<sup>[16]</sup>.

Lastly, the author would like to thank all the members of the EXAT Study Group for continued discussions on SDM-related technologies.

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